USING RENTAL SWAPS AND SALES TO MANAGE PORTFOLIO RISK AND TO FUND PROPERTY DEVELOPMENT

A. E. Baum[+]
C. J. Beardsley[*]
C. W. R. Ward[+]

Abstract

Swaps are private agreements between two companies to exchange cash flows in the future according to a prearranged formula. The first swap contracts in international capital markets were negotiated in 1981. Now, hundreds of billions of pounds of contracts are negotiated yearly on underlying assets ranging from commodities to equity indices, from fixed/floating deals to inverse floaters on foreign exchange transactions.

Cash flows from property in the form of rentals lend themselves readily to swaps analysis using derivatives pricing techniques that are now being applied to real estate in the US, UK and Australia.

However, the UK lease structure offers a particular opportunity to explore this means of analysis because of the complex option-like characteristics of the typical institutional lease.

Property cash flows can be readily 'partitioned' using derivatives pricing techniques with risk-neutral valuation techniques and Monte Carlo simulation. The resultant cash flow 'slices' can be swapped or sold both to enable portfolio diversification between funds and to provide funding for developers and corporate owners of property.

For swap transactions, payments are netted out at the end of the swap period with the property never being sold thus eliminating the high transaction costs associated with property transfers. In effect, a surrogate property sale can be synthetically structured with reversion belonging to the seller of the swap. For property developers or corporations seeking funding, parties can enter into future or forward contracts to sell the cash flows.

This paper examines pricing and market infrastructure issues surrounding the establishment of markets in both rental swaps and sales. The paper shows how these concepts might be applied to standard UK leases.

[+]Department of Land Management and Development,
The University of Reading,
Whiteknights,
Reading,
RG6 6AW,

*ISMA Centre, University of Reading
Whiteknights
Reading
RG6 6AW

Phone: +44 (0) 118 931 6301, Fax: +44 (0) 118 931 4741 E-mail:
c.j.beardsley@ismacentre.rdg.ac.uk

Introduction

In an earlier paper (Baum, Beardsley and Ward, 1999), we reviewed the sensitivity of derivatives pricing models of lease valuation to changes in commonly used parameters. We concluded in that paper that in the case of non-tradable property assets (non-tradable in the sense that direct property investments are not listed), it is necessary to estimate the 'market price of risk' of the property (see Hull, 1997 and Shimko, 1992). We may behave then as though the world were risk neutral in order to give a correct value for derivatives.

This paper furthers the concept of using Monte Carlo path dependent valuation techniques to price the embedded option element in the UK upward only lease and open market lease with respect to using techniques that are employed widely in global capital markets. We also use a mean reverting process to determine rental yields.

Specification of the valuation variables used in our model will require a paradigm shift on the part of valuers and investors away from the concept of one implied rental growth parameter to specification of two distinct processes viz. drift and volatility. Valuers and investors will also need to isolate the market price of risk (λ) and the expected market premium over the risk free rate to be able to value property derivatives properly in a risk neutral world.

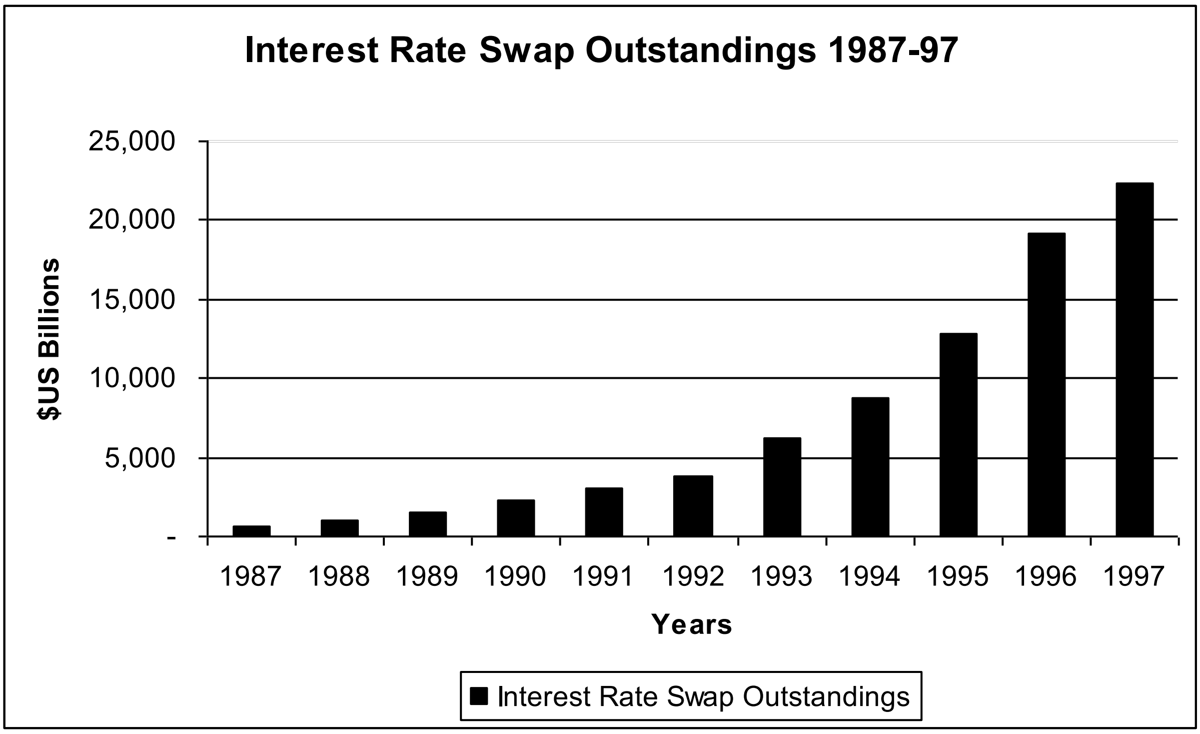
In the paper, we review the mechanisms and credit risk exposure for a 'plain vanilla' fixed for floating interest rate swap, and an equity swap. We show then how such structures might be applied to swapping property rentals using an example of a 25-year lease with (a) upward only review and (b) open market review subjected to differing drifts and volatilities explained in our model.

Growth of the Swap Market

The following table and graph show the staggering increase in interest rate swaps in the period 1987-97.

Table 1: Interest Rate Swap Outstandings ($US billions): 1987-1997

Calendar Year End	Interest Rate Swap Outstandings
1987	682.8
1988	1,010.2
1989	1,502.6
1990	2,311.5
1991	3,065.1
1992	3,850.8
1993	6,177.3
1994	8,815.6
1995	12,810.7
1996	19,170.9
1997	22,291.3
Source: International Swaps and Derivatives Association, Inc	



Source: International Swaps and Derivatives Association, Inc

<u>A Primer on Swaps</u>

Interest Rate Swaps

The most commonly used swap is an interest rate swap which is a bilateral contract committing each party to make payments to the other at regular intervals over an agreed period. Typically one party makes payment based on a fixed rate of interest throughout the contract period while the other party's payments are reset regularly to a floating reference rate such as London Interbank Offered Rate (LIBOR). If the two payment streams are in the same currency we have an interest rate swap; if the transaction is in different currencies we have a currency swap.

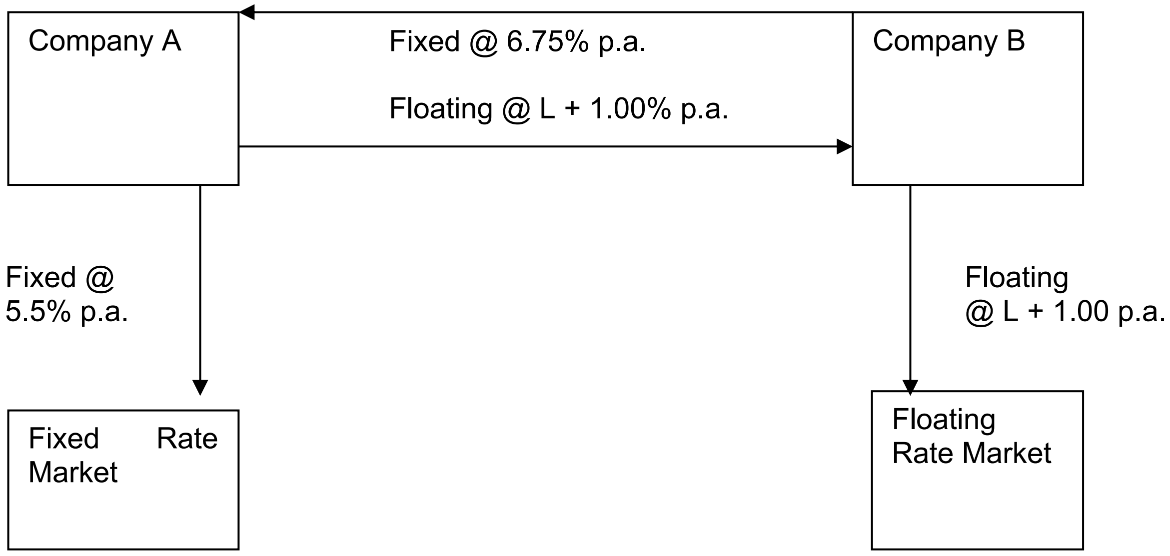
Swaps enable both parties to borrow in the most efficient way by separating the rate basis of a loan from the market or instrument in which the loan is raised.

Example

A and B are two companies each of which wishes to borrow £stg100 million. The funding characteristics of the two borrowers are as follows (T refers to the yield on Gilts and L to the current LIBOR rate)

Company A		Company B	
Amount:	£100 million	Amount:	£100 million
Period:	5 years	Period:	5 years
Requires:	Floating	Requires:	Fixed
Can issue at:		Can issue at:	
Fixed: 5.50%	T + 50 b.p.	Fixed: 7.00%	T + 200 b.p.
Floating:	L + 25 b.p.	Floating:	L + 100 b.p.

Without a swap A would borrow floating money and pay LIBOR + 0.25%, while B would borrow fixed and pay 7%. Instead, let A issue fixed rate debt and B issue floating. The two parties then agree that B will make fixed rate payments to A at 6.75% while A makes floating payments to B at LIBOR + 1.00%. The cash flows involved are shown below:



Cost: L + 1.00% + 5.50% - 6.75%
= L – 0.25% (saving 0.5%)

Cost: 6.75% + L + 1.00% – (L + 1.00)
= 6.75% (saving 0.25%)

Thanks to the swap both sides have saved money. A saved 0.5% p.a. while B saved 0.25% p.a. To see where this saving came from consider if the borrowers had accessed directly the markets quoting their required rate bases the total interest paid by the companies would have been:

7% + (L + 0.25%) = L + 7.25%

After the swap the total paid to the market was:

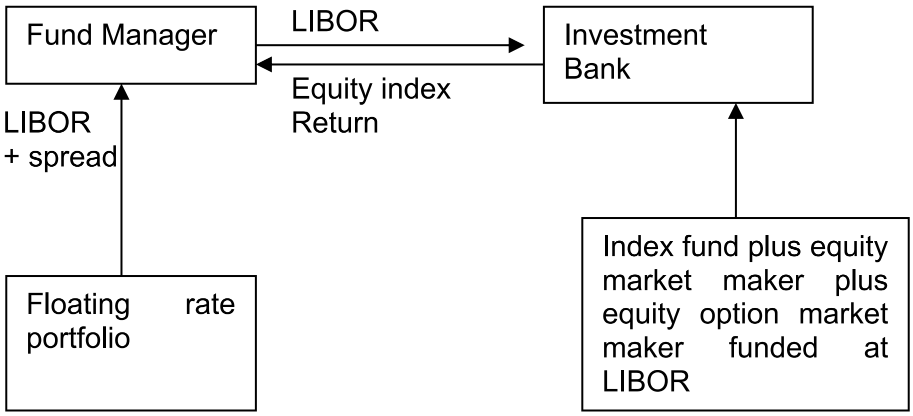
5.5% + (L + 100) = L + 6.5%

The saving was share 2/3:1/3 in favour of the stronger credit. The example above illustrates how the swap allows two parties to benefit if each exploits its comparative advantage in funding. Company A has an absolute advantage over company B in both the fixed and floating markets. However, it has a comparative advantage in the fixed rate market being able to borrow at 150 basis points less than B, whereas in the floating market the advantage is only 75 basis points.

Equity Swaps

With the growth in the interest rate and currency swap markets, other products, such as equity swaps, soon came along. Equity swaps involve fixed or floating rate interest related cash flows for the total rate of return (capital appreciation and dividends) of an equity index. Alternatively, the FTSE 100 index may be swapped for, say, the S & P 500. The subtle change in these types of swaps is that, compared to the interest rate swap discussed above, both parties are uncertain about the future cash flows. Equity swaps can be used in tactical asset allocation. Assume an equity investor wishes to reduce exposure to equity market A and increase exposure to equity market B. This can be achieved by entering into a swap to receive LIBOR and pay the returns of market A, and simultaneously entering into a swap to pay LIBOR and receive the returns to market B.

The figure below illustrates an equity swap diagrammatically:



In the above example, the fund manager has used the interest proceeds from a floating rate LIBOR portfolio to swap for the total return on an equity index in another market. The investment banking counter party has borrowed to deliver the index return either by straight purchase of the index future for the relevant holding period or by using market markers in the equity and option markets to create a synthetic index future. The costs of such transactions will be met from the LIBOR interest received.

For a useful discussion on swaps and relevant pricing issues in relation to swaps generally, see Watsham, 1998.

Credit risks associated with swaps

The swap mechanism involves:

- The notional exchange of principal amounts, and
- Netting out of relevant payments on settlement dates

Thus, counter-party credit exposure is minimised. If one side defaults on due date, the other side is not compelled to complete. Swaps can be structured also to account for mismatching of payment streams by holding funds with a third party 'stakeholder' until settlement.

Using the swaps mechanism, it is clearly possible to swap property rentals with an up front payment from one party to the other to account for the different drifts and volatilities inherent in the underlying property assets. No sale of the property is involved with the reversion residing with the seller of the relevant cash flows. Alternatively, if no swap counter-party can be found, the rental can be sold outright as an investment with or without uplifts.

We turn below to the method of pricing such transactions using Monte Carlo simulation plus mean reversion techniques. We then set out examples of possible transactions together with pricing parameters.

A Model for Pricing Property Rental Swaps using Monte Carlo Simulation and the Market Price of Risk

In our model, we use a discrete time analog of the Black-Scholes option-pricing model.

Let:

P_o = the price of property now
T = any future time (measured in years)
P_T = the price of the property at time T – it is a random variable and its value is not known until time T
Z = a standard normal variable (with mean = 0 and standard deviation = 1)
μ = Mean expected percentage growth rate of the property (per year) encompassing both rental yield and capital appreciation expressed as a decimal
σ = Standard deviation of the expected growth rate of the property (per year) expressed as a decimal.
λ = The market price of risk of the property (see below)

Using this notation, the future price of the property may be modelled as:

$$P_T = P_o * \exp[(\mu - \lambda - \frac{\sigma^2}{2})*T + \sigma * Z* \sqrt{T}\ ] \quad (1)$$

Market price of risk

The market price of risk in the above equation is reflected by the expected return on the asset (μ) being reduced to a level that is equivalent to the risk neutral return by subtracting from the expected return a premium (λ) that reflects the systematic risk of the non-tradable asset viz.

$$\begin{aligned} &\hat{\alpha}_x = \alpha_x - \beta(\alpha_m - r_f) \\ &\text{where} \\ &\alpha_x = \text{expected return on the asset} \\ &\alpha_m = \text{expected return on the market} \\ &\hat{\alpha}_x = \text{adjusted (risk neutral) return on the asset} \\ &\beta = \text{systematic risk component of the asset} \end{aligned} \quad (2)$$

This approach is similar in concept to transforming the cash flows into certainty equivalents (see, for example, Baum and Crosby, 1995) However, in equation (1), the expected return adjusted for the market price of risk must be further adjusted downwards by $\frac{\sigma^2}{2}$ to account for the Geometric Brownian Motion (GBM) process assumed for the property. For an intuitive explanation of GBM refer to our earlier paper (op. cit.) and Chriss (1997) Once all the adjustments are made, we may then discount the resultant cash flows by the risk free rate. Hull (1997) provides an excellent discussion on why this is so and a number of examples of using the market price of risk in a risk-neutral world

Using the FTA All Share Index to capture systematic risk

To capture the systematic risk of the property, we regressed the IPD Annual Property Index (returns) against the All Share Index (returns) for the period 1971-1998. We used the first differences of the series to achieve stationarity with the following results:

Multiple R	.46777834
R-Squared	.21881657

Adjusted R-Squared	.15088758			
Standard Error	13.126006			
Durbin-Watson	1.8661426			
	Beta	SEB	T	SIG T
SHARES	.11702839	.0461067	2.5382098	0.01837357
CONSTANT	-.72194132	2.7461083 .	-.2628962	0.79497129

While the calculation of property *betas* and the extent of the equity risk premium are both thorny subjects in property research, for the purposes of our working calculations below, we assumed a *beta* of 0.12. The excess of share returns over the expected risk free rate was assumed to be 10%. Thus, our value for Lambda is:

λ = 0.12*(10%)

λ = 1.2%

<u>*Modelling the property*</u>

Equation (1) above represents two combined processes. The first process is known as the 'drift' [(μ - $\lambda - \frac{\sigma^2}{2}$)*T] that is a deterministic process combining all of the inputs of the model save for the random (0,1) normal variable. The drift represents the expected return from both the income and capital appreciation from the underlying property adjusted by the market price of risk (λ).

The second part of the process **(**σ * Z* $\sqrt{T}$**)** introduces uncertainty regarding the volatility of the total returns of the property. Combining these processes results in Geometric Brownian Motion with path dependency from using Monte Carlo simulation. The property is subjected to a constant value adjustment from a fixed source (the drift) and is buffeted by random jolts scaled by the volatility times the square root of time. This process is repeated many thousands of times until the amount of change in the statistics becomes less and less and meet a required minimum threshold (in our case the mean and standard deviation of results were less than 0.5%).

There are a number of subjective inputs into using equations (1) – (2) above, notably:

1. The choice of how high the relevant risk premium should be, and
2. The expected drift and volatility

Derivatives are by their nature complex. The above valuation procedure represents a paradigm shift from current valuation techniques. The investor or valuer has to specify the expected drift and volatility of the underlying property together with the market price of risk and risk premium of the market. Each property is considered a unique bundle of cash flow characteristics, which need to be part of the same risk free world for comparison purposes.

<u>Rent Reversion Model</u>

In our model, we also assume that the initial rental yield is known but does not remain constant. In practice, we observe a term structure of yields that implies changing rates over time. We have assumed a model based on Cox, Ingersoll and Ross (1985) in which the yield is mean reverting to an average rate of 7% per annum with a standard deviation of 0.7% per annum (based on the IPD UK Annual Index 1981-1998) The Cox, Ingersoll and Ross model specifies the mean reverting process as the following:

$$dr = a(b - r)dt + \sigma\sqrt{r}dz$$

where a, b, and σ are constants. This means that *r* follows geometric Brownian motion. The process has a constant expected growth rate of *a*(b-r)* where the short rate is pulled to a level *b* at rate *a*. Superimposed on this "pull" is a normally distributed stochastic term that has a standard deviation proportional to $\sqrt{r}$. This means that as the short-term interest rate increases, its standard deviation increases. The parameter, a, is kept constant at 7% imparting a 'stickiness' to the series.

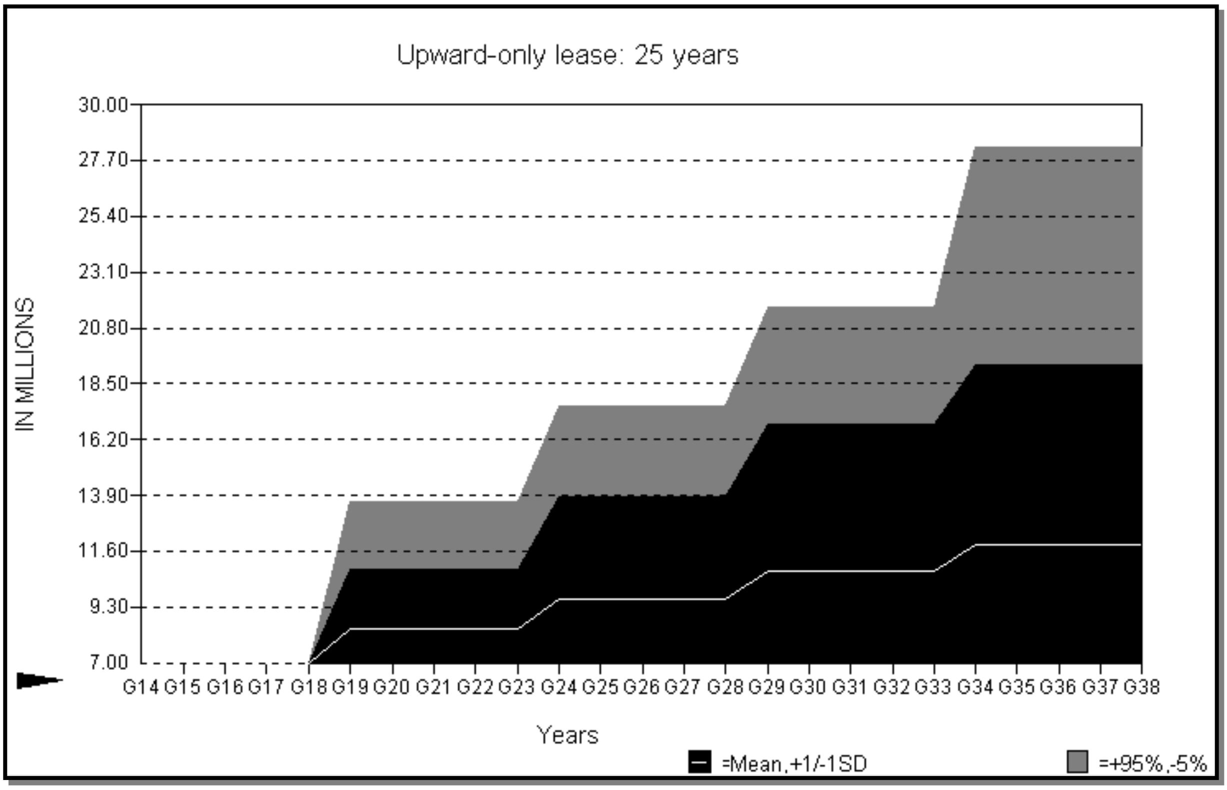
A Graphical Representation of the Model

Graphically, the model is shown below with nominal cash flows shown for:

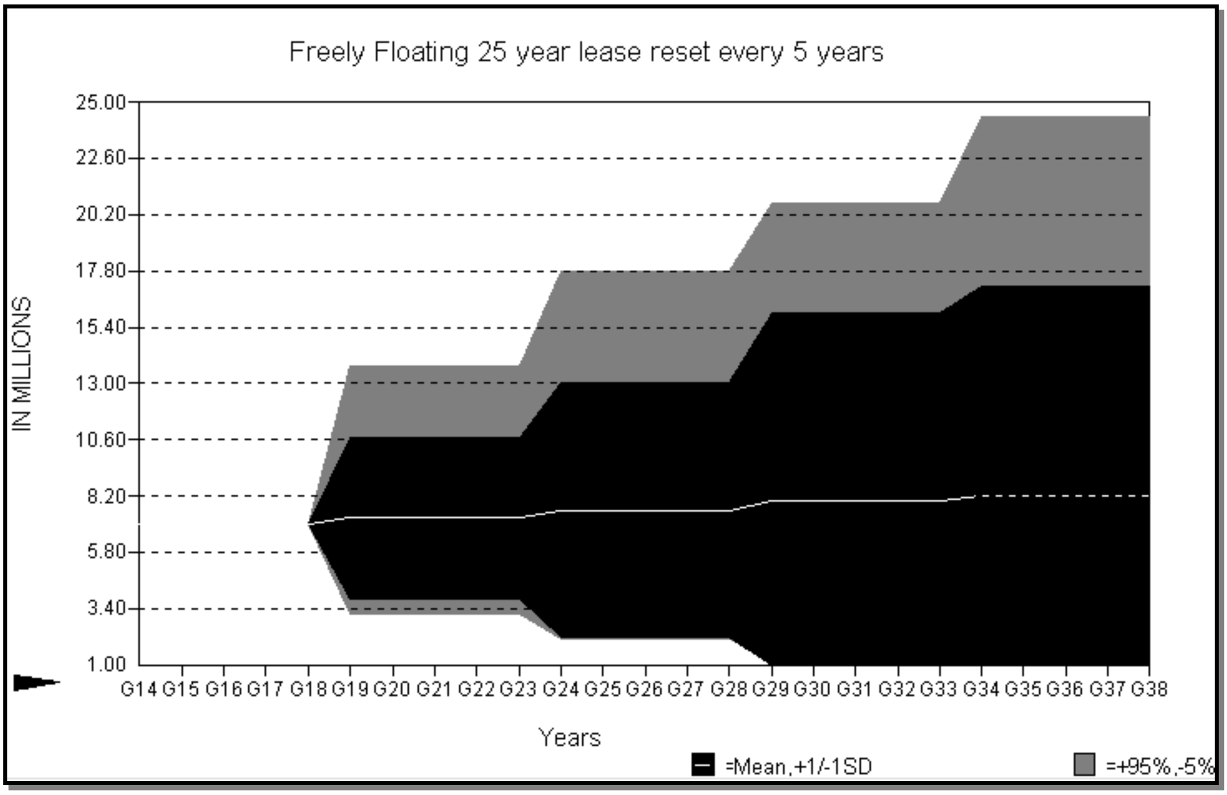
1. An upward only 25-year lease with 5 yearly reviews
2. A freely floating 25-year lease with 5 yearly reviews

In both cases, the examples use a property with a current value (P_0) of £100 million, a risk-free rate of 5% per annum, rent reversion as described above, a Lambda (λ) of 1.2% p.a., drift of 2% p.a. and volatility of 20% p.a. The graphs clearly show the incremental value in the upwards only lease with the expected value of the rentals (the white line) not falling below £7 million and climbing at a much steeper trajectory. In contrast, the freely floating lease rentals may decline in value throughout the 25-year period. The black area represents the value of the mean ± one standard deviation with the grey area showing the 95% confidence interval for the mean.

An upward only 25-year lease with 5 yearly reviews



A freely floating 25-year lease with 5 yearly reviews



Examples of Pricing

We offer some indicative pricing parameters based on the above model for:

- Swapping five year upward only on one property for five year freely floating on another property with the same drift and the same volatility
- Swapping five year upward only on one property for five year upward only on another property with different drifts and different volatilities
- Swapping five year upward only on one property for five year upward only on another property with the same drift but different volatilities

In all cases, the examples use a property with a current value (P_0) of £100 million, a risk-free rate of 5% per annum, rent reversion as described above and a Lambda (λ) of 1.2% p.a.

The examples are based on swaps only. Outright sale of expected rentals may be priced in the same fashion. In addition, the upward only lease by its very structure produces an annuity strip for the lifetime of the lease which can be thought of as a series of zero coupon bonds that are capable of being securitised, credit enhanced, and sold into the fixed income market. The annuity strip for the open market lease can be securitised for separate five yearly tranches after review.

Swapping five year upward only on one property for five year open market on another property with the same drift and the same volatility

The discounted cash flows for the upward only lease are shown in the first box cut into five-yearly ‘slices’. Discounted cash flows for the fully floating lease are treated similarly in the second box. It is evident that the upward only cash flows are far more valuable. If a swap can be arranged the owner of the freely floating cash flows would have to pay the net difference to the owner of the upward only cash flows (shown in the third box) for drift of 2% p.a. and volatility of 20% p.a.

Upward-only 25 year lease			Fourth Slice Years 21-25 £ 1,759,255	Total Fourth Slice £ 1,759,255
		Third Slice Years 16-20 £ 2,589,919	Third Slice Years 21-25 £ 2,017,031	Total Third Slice £ 4,606,949
	Second Slice Years 11-15 £ 3,239,218	Second Slice Years 16-20 £ 2,522,706	Second Slice Years 21-25 £ 1,964,685	Total Second Slice £ 7,726,609
First Slice Years 6-10 £ 4,972,613	First Slice Years 11-15 £ 3,872,675	First Slice Years 16-20 £ 3,016,042	First Slice Years 21-25 £ 2,348,896	Total First Slice £14,210,227

MINUS

Freely floating 25 year lease			Fourth Slice Years 21-25 £ 390,112	Total Fourth Slice £ 390,112
		Third Slice Years 16-20 £ 800,833	Third Slice Years 21-25 £ 623,689	Total Third Slice £ 1,424,522
	Second Slice Years 11-15 £ 832,033	Second Slice Years 16-20 £ 647,988	Second Slice Years 21-25 £ 504,653	Total Second Slice £ 1,984,673
First Slice Years 6-10 £ 929,762	First Slice Years 11-15 £ 724,099	First Slice Years 16-20 £ 563,929	First Slice Years 21-25 £ 439,188	Total First Slice £ 2,656,978

EQUALS NET PAYMENTS FLOATING TO UPWARD ONLY OF

Net Payments from fully floating to upward-only			Fourth Slice Years 21-25 £ 1,369,142	Total Fourth Slice £ 1,369,142
		Third Slice Years 16-20 £ 1,789,086	Third Slice Years 21-25 £ 1,393,342	Total Third Slice £ 3,182,428
	Second Slice Years 11-15 £ 2,407,185	Second Slice Years 16-20 £ 1,874,718	Second Slice Years 21-25 £ 1,460,032	Total Second Slice £ 5,741,935
First Slice Years 6-10 £ 4,042,852	First Slice Years 11-15 £ 3,148,576	First Slice Years 16-20 £ 2,452,114	First Slice Years 21-25 £ 1,909,708	Total First Slice £11,553,249

Swapping five year upward only on one property for five year upward only on another property with different drifts and volatilities

In the example above of a swap involving fully floating to upward only, the net differences are relatively high reflecting the probabilities that the fully floating lease may decline in rental value while the upward only lease cannot fall below its previous rental value.

It is, of course, possible to swap two upward only leases with different drifts and volatilities that might be attractive to funds wishing to diversify geographically by, say, retail property. In the example below, we have structured a swap with one property assumed to drift at 1% p.a. with a volatility of 10% p.a. while the other property drifts at 2% with a volatility of 20% p.a.

Upward-only 25 year lease: 2% drift and 20% volatility				
			Fourth Slice Years 21-25 £ 1,759,255	Total Fourth Slice £ 1,759,255
		Third Slice Years 16-20 £ 2,589,919	Third Slice Years 21-25 £ 2,017,031	Total Third Slice £ 4,606,949
	Second Slice Years 11-15 £ 3,239,218	Second Slice Years 16-20 £ 2,522,706	Second Slice Years 21-25 £ 1,964,685	Total Second Slice £ 7,726,609
First Slice Years 6-10 £ 4,972,613	First Slice Years 11-15 £ 3,872,675	First Slice Years 16-20 £ 3,016,042	First Slice Years 21-25 £ 2,348,896	Total First Slice £14,210,227

<u>MINUS</u>

Upward-only 25 year lease: 1% drift and 10% volatility				
			Fourth Slice Years 21-25 £ 512,797	Total Fourth Slice £ 512,797
		Third Slice Years 16-20 £ 809,810	Third Slice Years 21-25 £ 630,680	Total Third Slice £ 1,440,490
	Second Slice Years 11-15 £ 1,174,129	Second Slice Years 16-20 £ 914,413	Second Slice Years 21-25 £ 712,145	Total Second Slice £ 2,800,688
First Slice Years 6-10 £ 2,106,694	First Slice Years 11-15 £ 1,640,695	First Slice Years 16-20 £ 1,277,774	First Slice Years 21-25 £ 995,132	Total First Slice £ 6,020,295

<u>EQUALS NET PAYMENTS OF</u>

Net Payments				
			Fourth Slice Years 21-25 £ 1,246,458	Total Fourth Slice £ 1,246,458
		Third Slice Years 16-20 £ 1,780,109	Third Slice Years 21-25 £ 1,386,350	Total Third Slice £ 3,166,459
	Second Slice Years 11-15 £ 2,065,089	Second Slice Years 16-20 £ 1,608,293	Second Slice Years 21-25 £ 1,252,540	Total Second Slice £ 4,925,921
First Slice Years 6-10 £ 2,865,920	First Slice Years 11-15 £ 2,231,980	First Slice Years 16-20 £ 1,738,268	First Slice Years 21-25 £ 1,353,765	Total First Slice £ 8,189,932

The advantage of the lease with higher drift and volatility is self-evident.

<u>Swapping five year upward only on one property for five year upward only on another property with the same drift but different volatilities</u>

The impact of different volatilities on the model can be assessed by modelling an upward only lease with the same drift (1% p.a.) but with different volatilities (10% p.a. and 20% p.a.)

Upward-only 25 year lease: 1% drift and 20% volatility			Fourth Slice Years 21-25 £ 1,169,122	Total Fourth Slice £ 1,169,122
		Third Slice Years 16-20 £ 1,776,280	Third Slice Years 21-25 £ 1,383,368	Total Third Slice £ 3,159,648
	Second Slice Years 11-15 £ 2,449,119	Second Slice Years 16-20 £ 1,907,376	Second Slice Years 21-25 £ 1,485,466	Total Second Slice £ 5,841,961
First Slice Years 6-10 £ 4,200,875	First Slice Years 11-15 £ 3,271,645	First Slice Years 16-20 £ 2,547,960	First Slice Years 21-25 £ 1,984,353	Total First Slice £12,004,833

<u>MINUS</u>

Upward-only 25 year lease: 1% drift and 10% volatility			Fourth Slice Years 21-25 £ 512,797	Total Fourth Slice £ 512,797
		Third Slice Years 16-20 £ 809,810	Third Slice Years 21-25 £ 630,680	Total Third Slice £ 1,440,490
	Second Slice Years 11-15 £ 1,174,129	Second Slice Years 16-20 £ 914,413	Second Slice Years 21-25 £ 712,145	Total Second Slice £ 2,800,688
First Slice Years 6-10 £ 2,106,694	First Slice Years 11-15 £ 1,640,695	First Slice Years 16-20 £ 1,277,774	First Slice Years 21-25 £ 995,132	Total First Slice £ 6,020,295

<u>EQUALS NET PAYMENTS OF</u>

			Fourth Slice Years 21-25 £ 656,325	Total Fourth Slice £ 656,325
		Third Slice Years 16-20 £ 966,470	Third Slice Years 21-25 £ 752,688	Total Third Slice £ 1,719,158
	Second Slice Years 11-15 £ 1,274,990	Second Slice Years 16-20 £ 992,963	Second Slice Years 21-25 £ 773,321	Total Second Slice £ 3,041,273
First Slice Years 6-10 £ 2,094,181	First Slice Years 11-15 £ 1,630,950	First Slice Years 16-20 £ 1,270,186	First Slice Years 21-25 £ 989,221	Total First Slice £ 5,984,538

The advantage of the lease with higher volatility is self-evident.

Conclusions

Property combines features both from the fixed income market and from the equity market. We have shown in this paper how it is possible to value and to trade in the fixed income and equity rental derivatives of property without having to sell the underlying physical.

In the absence of the ability to eliminate Lambda (λ) thus reducing μ to the risk-free rate by fully hedging the property; we believe our model offers property investors, developers and owners an opportunity to value diversification of portfolio risk plus the ability to 'bet' on the valuation of rental derivatives without the need to sell the underlying physical property.

The markets for strips, swaps and sales of property rentals would include:

- Corporate owners of property seeking to release cash flow
- Developers wishing to sell forward future rentals to release cash
- Investors seeking long term fixed rate investments
- Funds wishing to swap risk to diversify their portfolio

Analytical tools necessary to accomplish such valuations are necessarily more complex than those tools currently used in the property market. We do not believe this should represent an impediment to using what are now considered standard techniques developed in other areas of the capital markets that can be readily applied to the property area.

****

Bibliography

Bachelier, L., 1900: Theory of speculation (English translation), in : Cootner, P., ed., The random character of stock price market prices, MIT Press, Cambridge, 497 - 505.

Baum, A.E., Beardsley, C.J., and Ward, C.W.R., 1999, *'Derivatives Pricing Approaches to Valuation Models: Sensitivity Analysis of Underlying Factors'* The European Real Estate Society, Sixth European Conference, Athens, Greece, 22-25 June 1999

Baum A. E. & Crosby N, 1995, *Property Investment Appraisal- 2nd Edition* International Thompson Publishing

Black, F. and Scholes, M., 1972: The valuation of option contracts and a test of market efficiency, Papers and proceedings of the Thirtieth Annual Meeting of the American Finance Association, 27 - 29 Dec. 1971, Journal of Finance, 27, 399 - 417.

Black, F. and Scholes, M., 1973: The Pricing of Options and Corporate Liabilities, Journal of Political Economy, 3, 637 - 654.

Boness, A., 1964: Elements of a theory of stock-option value, Journal of Political Economy, 72, 163 - 175.

Boyle, P., 1976: Options: A Monte Carlo Approach, Journal of Financial Economics, 4, 323 - 338.

Capozza, D. & Sick, G., 1991: Valuing Long-Term Leases: The Option to Redevelop, Journal of Real Estate Finance and Economics, 4, 209 - 223.

Capozza, D. & Sick, G., 1994: The Risk Structure of Land Markets, Journal of Urban Economics, 35, 297 - 319.

Cappozza, D. & Li, S., 1994: The Intensity and Timing of Investment: The Case of Land, The American Economic Review, Vol. 84, No. 4, 889 - 904.

Chidambaran, N. & Figlewski, S., 1995: Streamlining Monte-Carlo Simulation with the Quasi-Analytic Method, An Analysis of a Path-Dependant Option Strategy. Unpublished working paper

Chriss N (1997*) Black-Scholes and Beyond – Option Pricing Models* Irwin

Cox J, Ingersoll J & Ross S (1985) 'A Theory of the Term Structure of Interest Rates' Econometrica 53 pp. 385-407

Cox, C., Ross, S. And Rubinstein, M., 1979: Option Pricing: A Simplified Approach, Journal of Financial Economics, 7, 229 - 263.

Dixit, A. & Pindyck, R., 1994: Investment Under Uncertainty, Princeton.

French N. S. and Ward C W R, "Valuation and Arbitrage", Journal of Property Research, Vol. 12, 1, Spring 1995, pp 1-11

French, N. S. and Ward, C.W.R, "Applications of the Arbitrage Method of Valuation", Journal of Property Research**,** Vol. 13, 1, 1996, 47 - 56, Galai, D., 1975: Pricing options and the efficiency of the Chicago Board Options Exchange, unpublished Ph.D. dissertation (University of Chicago, Chicago, I11.).

Geltner, D. & Riddiough, T., 1996: Insights on the Effect of Land Use Choice: The Perpetual Option on the Best of Two Underlying Assets, 39, 20 - 50.

Grenadier, S, 1995: Valuing lease contracts: a real- options approach. Journal of Financial Economics, 38, 297-331

Hull, John C. (1997) *Options, futures, and other derivatives* Prentice-Hall International, London

Investment Property Databank, IPD annual index 1998

Lee, S.L. and Ward, C.W.R, Estimating the volatility of individual property returns, Working paper, Department of Land Management and Development, University of Reading, June, 1999.

Quigg, L., 1993: Empirical Testing of Real Option-Pricing Models, The Journal of Finance, Vol. XLVIII, No. 2, June, 621 - 640.

Quigg, L., 1995: Optimal land development, in : Trigeorgis, L., ed., Real Options in Capital Investments, Praeger, London.

Samuelson, P., 1965: Rational theory of warrant pricing, Industrial Management Review, 6, 13 - 31.

Shimko, David C. (1992) *Finance in Continuous Time – A Primer* Kolb Publishing Company

Sprenkle, C., 1964: Warrant prices as indicators of expectations and preferences, in: Cootner, P., ed., The random character of stock price market prices, MIT Press, Cambridge, 412 - 474.

Titman, S., 1985: Urban land prices under uncertainty, American Economic Review, 75, 505 - 514.

Ward, C. and French, N. The Valuation of Upwards Only Rent Reviews: an Option Pricing model, Journal of Property Valuation & Investment, 15(2), 1997, 171-182

Ward, C., Hendershott, P. and French, N. Pricing Upwards Only Rent Review Clauses: An International Perspective. Journal of Property Valuation & Investment, 16(5), 1998, 447-454.

Watsham, Terry J. (1998) *Futures and Options in Risk Management: Second Edition* Thomson Business Press

Williams, J., 1991: Real Estate Development as an Option, Journal of Real Estate Finance and Economics, June, 4(2), 191 - 208.